

Rewinding a bit..... To yesterday

## We have detection histories – now what?

$\Pr(eh = 0 0 1 0 1 1 0 0) =$

$(1-\epsilon_3)(1-p_4) (1-\epsilon_4)p_5 (1-\epsilon_5)p_6 [\epsilon_6 + (1-\epsilon_6)(1-p_7)\{\epsilon_7+(1-\epsilon_7)(1-p_8)\}] = H_1$

Likelihood (parameters | data) =  $H_1^{\text{No. cases}}$   $H_2^{\text{No. cases}}$   $H_x^{\text{No. cases}}$

0001010010	10
0010001101	3
0101010100	4
1001000000	18
0001010111	2
0011101000	1
	3
	No. cases

**Maximize likelihood function so that, under the assumed statistical model, the observed data is most probable.**

[https://en.wikipedia.org/wiki/Maximum\\_likelihood\\_estimation](https://en.wikipedia.org/wiki/Maximum_likelihood_estimation)

Likelihood (parameters | data) =  $H_1^{\text{No. cases}}$   $H_2^{\text{No. cases}}$   $H_x^{\text{No. cases}}$

Maximize **likelihood function** so that, under the assumed statistical model, the observed data is most probable.

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[https://en.wikipedia.org/wiki/Maximum\\_likelihood\\_estimation](https://en.wikipedia.org/wiki/Maximum_likelihood_estimation)

Likelihood (parameters | data) =  $H_1^{\text{No. cases}}$   $H_2^{\text{No. cases}}$   $H_x^{\text{No. cases}}$

- $\epsilon(\text{constant})p(\text{time-varying})$
- $\epsilon(\text{time-varying})p(\text{sea-levels})$

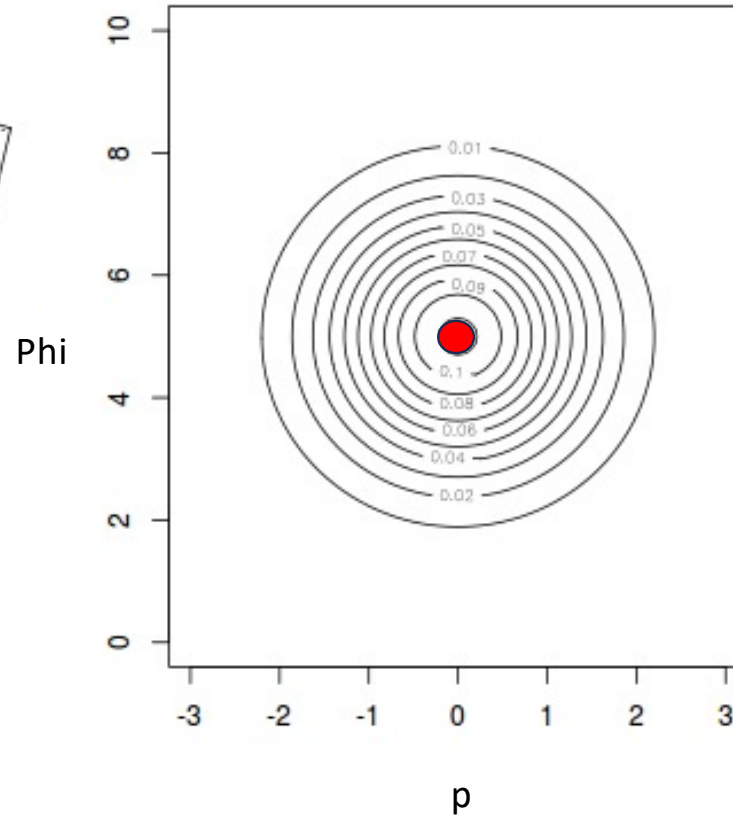
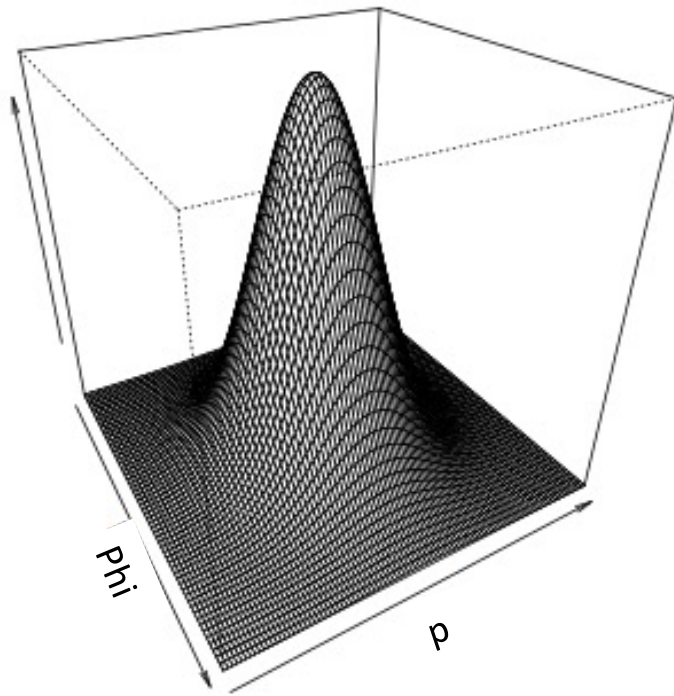
Maximize **likelihood function** so that, under the **assumed statistical model**, the **observed data** is **most probable**.

[https://en.wikipedia.org/wiki/Maximum\\_likelihood\\_estimation](https://en.wikipedia.org/wiki/Maximum_likelihood_estimation)

$$\text{Likelihood (parameters | data)} = H_1^{\text{No. cases}} H_2^{\text{No. cases}} H_x^{\text{No. cases}}$$

- $\epsilon(\text{constant})p(\text{time-varying})$
- $\epsilon(\text{time-varying})p(\text{sea-levels})$
- $\phi(\text{constnat})p(\text{sconstant})$

0001010010  
0010001101  
0101010100  
1001000000  
0001010111  
0011101000



<https://www.flutterbys.com.au/stats/tut/tut4.3.html>

```

> AIC(dipper.const,dipper.phi.t, dipper.p.t, dipper.phi.t.p.t)
      model npar rank   logLik   AIC   AICc   dAIC  AICwt
dipper.const  p~1 phi~1    2     2 -333.4188 670.838 671.068  0.000 0.7953
dipper.phi.t  p~1 phi~t    7     7 -329.8650 673.730 676.113  2.892 0.1873
dipper.p.t    p~t phi~1    7     7 -332.2401 678.480 680.863  7.642 0.0174
dipper.phi.t.p.t p~t phi~t   12    12 -328.4751 680.950 688.379 10.112 0.0000
>

```

Higher log-likelihood value better fit to the data

Least negative (highest) is dipper.phi.t.p.t

Why is dipper.constant on “top” (the code rearranges the models with the best as the first row)

$$AIC = 2k - 2 \ln \hat{L}$$

$$AICc = AIC + \frac{2k(k+1)}{n-k-1}$$

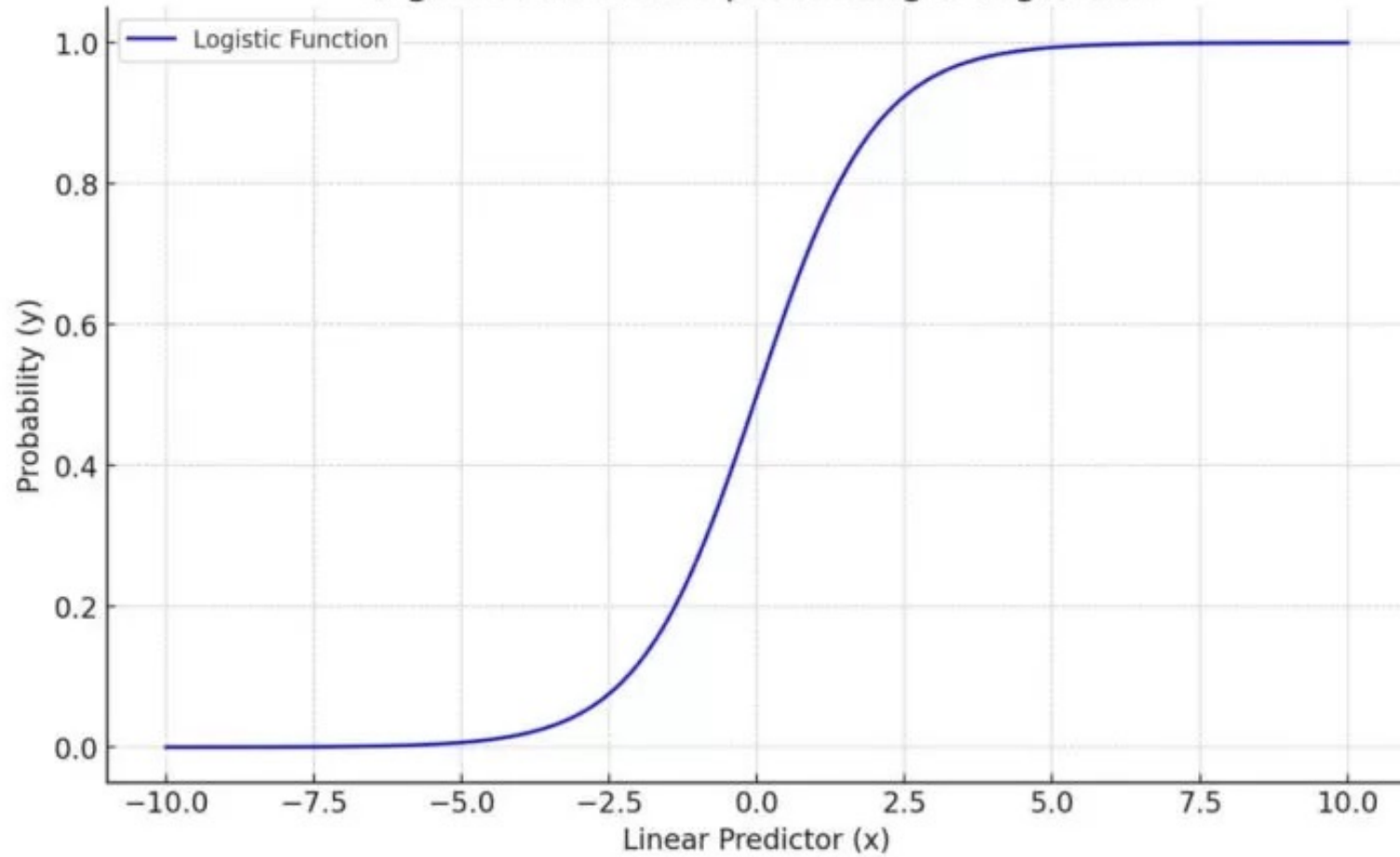


## Covariate modeling via link functions

- Link functions allows model to accommodate response variables that do not naturally fit a linear scale, such as probabilities between 0 and 1
- In logistic regression model (one example of a GLM Generalized Linear Model ), the logit link function transforms the probability scale to an unbounded scale, where linear regression can be applied
- **logit link function is ideal for binary outcome modelling (survive or not survive, preserved or not preserved)**
- **correct link function improves model fit and accuracy**
- Identity, Probit, Log , Inverse, Sine are other link functions you might encounter.

<https://statisticseasily.com/link-functions-in-generalized-linear-models/>

Logistic Function representing a Logit Link



## Covariate modeling via link functions

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$

Time specific covariates

- Duration of bin
- Sea-level
- Temperature

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \beta_0$$

`invlogit(2.2262629)`

`[1] 0.9025833`

`invlogit(0.2421485)`

`[1] 0.560243`

```

R 4.4.1 . ~/
p      logit
phi    logit

Beta parameters (coefficients)
      beta SE.beta   lcl   ucl
p  2.2262629 0.3251461 1.58898831 2.863537
phi 0.2421485 0.1020139 0.04220503 0.442092

Eigenvalues : 1 0.09066
Numerical rank of Hessian : 2 (svtol = 1e-05)

Variance-covariance matrix of beta parameters
      p      phi
p  0.105719960 -0.008508673
phi -0.008508673  0.010406826

Fitted (real) parameters evaluated at base levels of covariates

p
session estimate SE.estimate   lcl   ucl
1981      NA      NA      NA      NA
1982 0.9025833  0.02858903 0.8304737 0.9460142
1983 0.9025833  0.02858903 0.8304737 0.9460142
1984 0.9025833  0.02858903 0.8304737 0.9460142
1985 0.9025833  0.02858903 0.8304737 0.9460142
1986 0.9025833  0.02858903 0.8304737 0.9460142
1987 0.9025833  0.02858903 0.8304737 0.9460142

phi
session estimate SE.estimate   lcl   ucl
1981 0.560243  0.02513323 0.5105497 0.6087574
1982 0.560243  0.02513323 0.5105497 0.6087574
1983 0.560243  0.02513323 0.5105497 0.6087574
1984 0.560243  0.02513323 0.5105497 0.6087574
1985 0.560243  0.02513323 0.5105497 0.6087574
1986 0.560243  0.02513323 0.5105497 0.6087574
1987      NA      NA      NA      NA
  
```

## Covariate modeling via link functions

$$\text{logit}(\varepsilon_{i,t}) = \log\left(\frac{\varepsilon_{i,t}}{1 - \varepsilon_{i,t}}\right) = \beta_0 + \beta_1 x_i + \beta_2 y_t$$

$$\varepsilon_{i,t} = \frac{e^{\beta_0 + \beta_1 x_i + \beta_2 y_t}}{1 + e^{\beta_0 + \beta_1 x_i + \beta_2 y_t}}$$

# A bit of everything related to CMR in paleo

Lee Hsiang Liow

Natural History Museum and Centre for Planetary Habitability, University of Oslo, Norway

16.8.2024 (Friday)



# Summary

$p \neq 1$

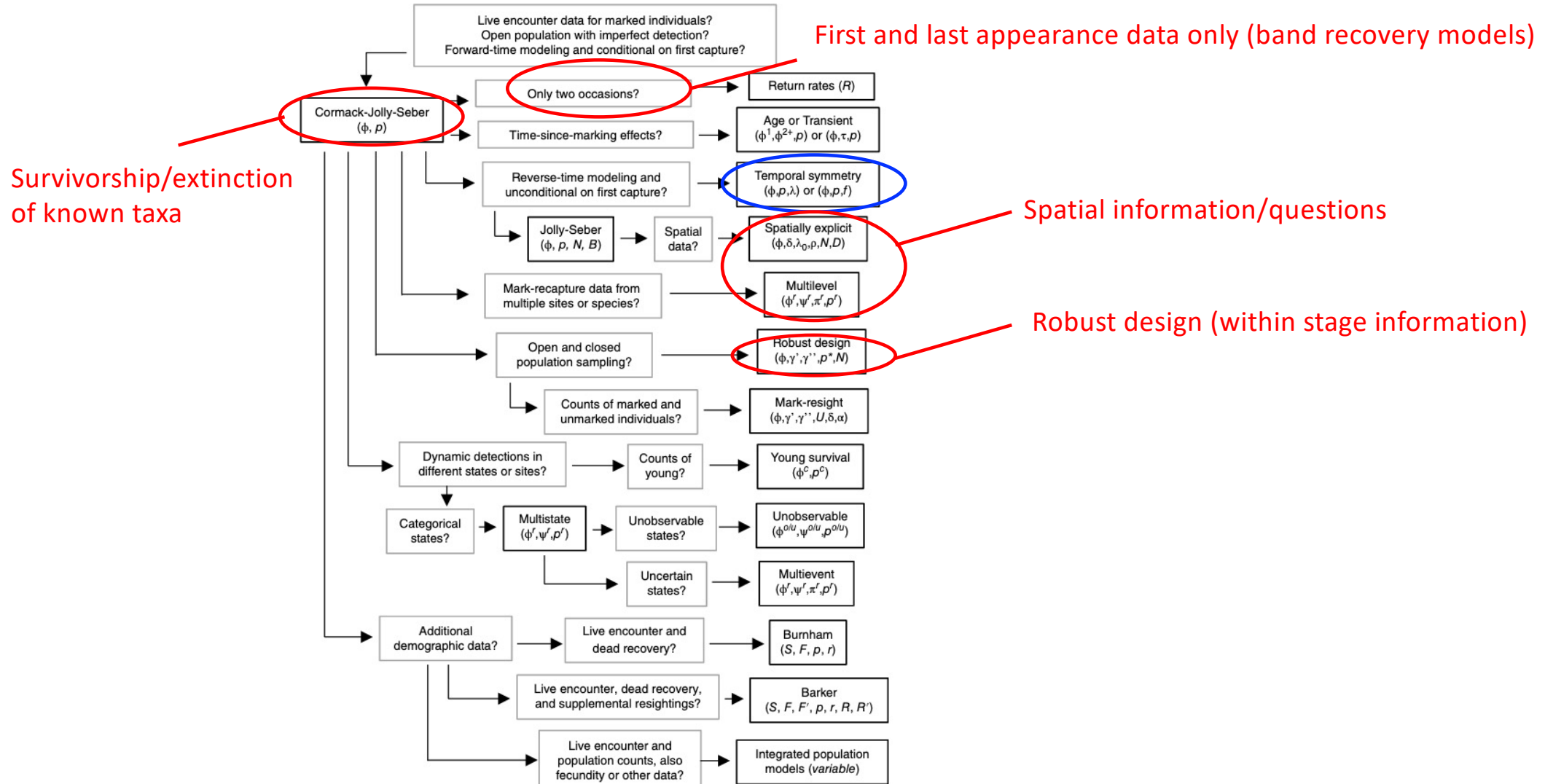
Two types of zeros

# Common tutorial questions

- Additive versus multiplicative models
- Model-averaging



# Many models: few explored in paleo



MARK is the “main” software and easy to read reference (but you must “translate”)



The screenshot shows a web browser window displaying the 'program MARK' website. The page has a blue header with the title 'program MARK' and the subtitle 'A Gentle Introduction'. Below the header is a green navigation bar with links for 'Home', 'About', 'Download', 'Support Forum', and 'Training'. The main content area is divided into two columns. The left column features a book cover for 'Program MARK - a gentle introduction - (edition 3.2)' by e.g. cooch & g.c. white (eds). Below the cover is a 'Book chapters & data files' section with a 'Select chapter' dropdown menu. The right column has an 'Overview' section with text describing the program, followed by an 'About the book...' section with more details. A 'Thoughts for the day...' section is also visible at the bottom left.

<http://www.phidot.org/software/mark/docs/book/>

Gary White

# Self learning material and software

Gary White MARK

<http://www.phidot.org/software/mark/>

(great if you are windows user, a bit more involved if you us Mac) great to pair with Rmark (Jeff Laake)

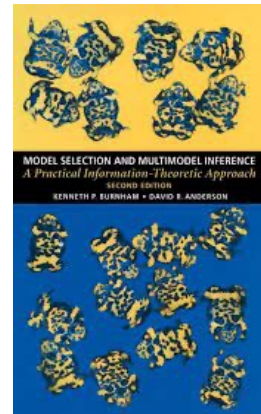
Gary White MARK book

<http://www.phidot.org/software/mark/docs/book/>

Michael Conroy lecture notes (bare bones description of models; useful for Mark/Rmark code and data)  
<https://sites.google.com/site/cmsoftware/>

Burnham and Anderson 2022:

**Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach**



# R packages for CMR

- [RMark](#) (runs MARK from R, works best with Windows)
- [serc](#) and [openCR](#)
- [Marked](#) (some overlap with openCR and MARK but this doesn't have Pradel models)
- [R2ucare](#) (for goodness of fit tests and simulations for CMR)
- [mra](#), [Rcapture](#), [BaSTA](#)

# Other languages for capture recapture

**Python** <https://www.python.org/>

- <https://austinrochford.com/posts/2018-01-31-capture-recapture.html>
- [https://pyro.ai/examples/capture\\_recapture.html](https://pyro.ai/examples/capture_recapture.html)

**Stan** <https://mc-stan.org/>

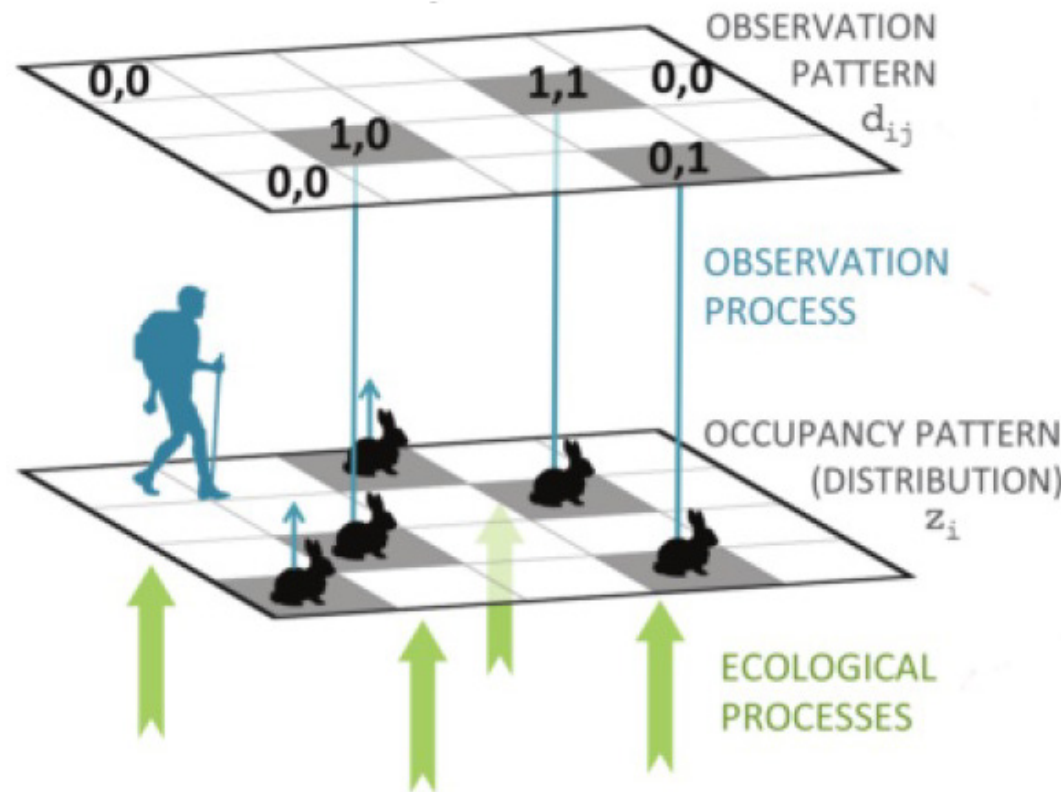
- <https://mc-stan.org/docs/stan-users-guide/latent-discrete.html#mark-recapture-models>

**BUGS, JAGS**

<https://bcss.org.my/tut/bayes-with-jags-a-tutorial-for-wildlife-researchers/abundance-from-capture-recapture-data/basic-spatial-capture-recapture-models/>

<https://esajournals.onlinelibrary.wiley.com/doi/full/10.1002/ecs2.3810>

# Site-occupancy modeling in statistical ecology

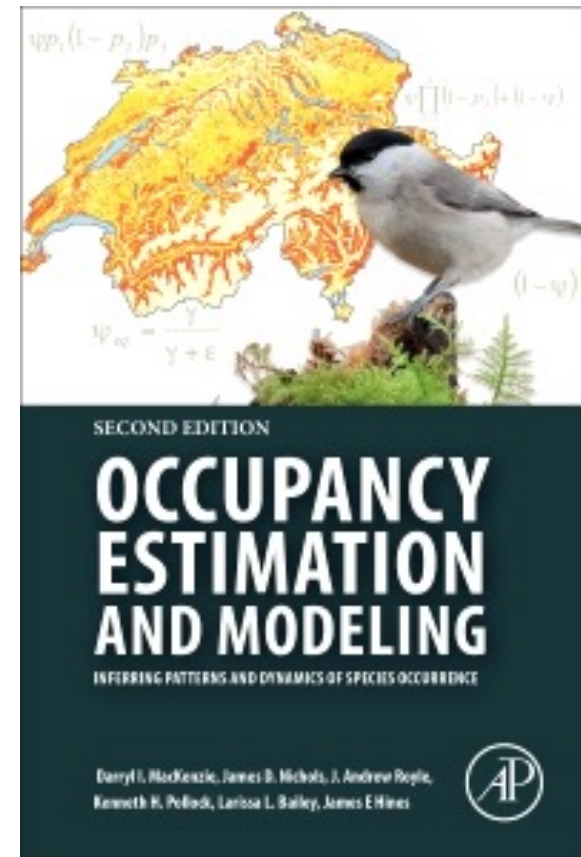


Not to be confused with  
Foote, M. *et al.* (2007) Rise and fall of species occupancy  
in Cenozoic fossil molluscs. *Science* (80- ). 318, 1131–11341

Foote, M. (2016) On the measurement of occupancy in  
ecology and paleontology. *Am. Nat.* 42, 707–729

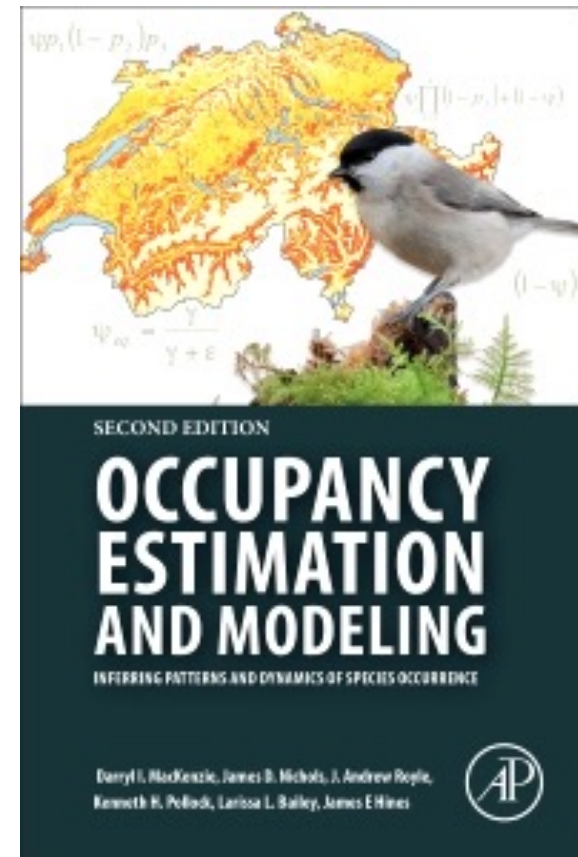
<http://www.seec.uct.ac.za/single-season-occupancy-models-using-bayesian-approach>

# (Site)-Occupancy modeling



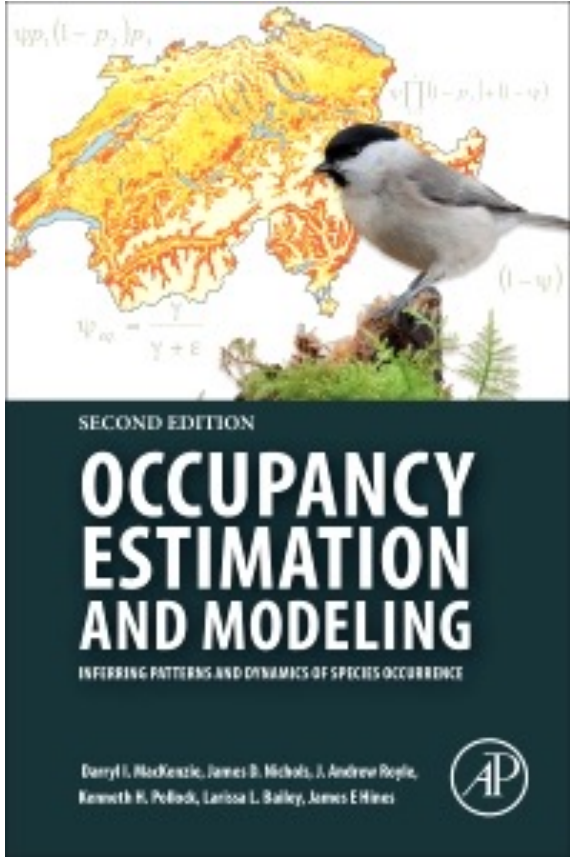
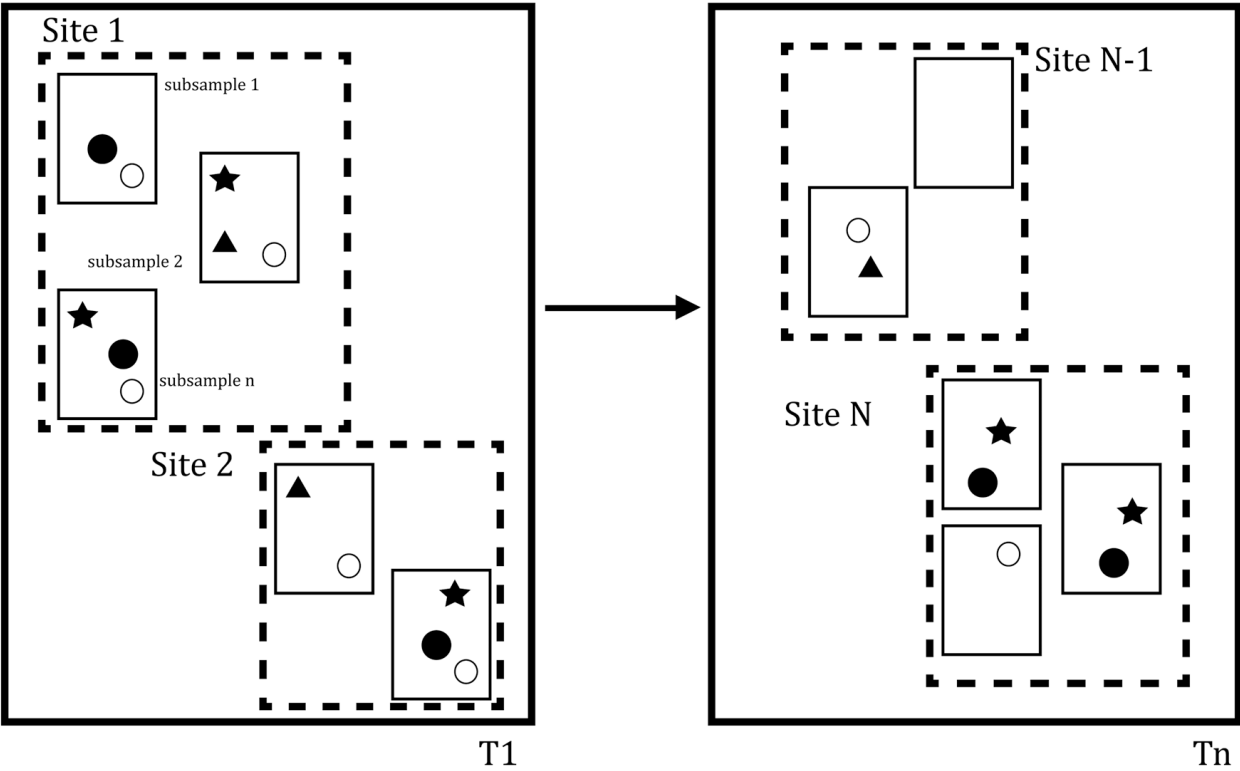
# Occupancy modeling in paleo

- Liow, L.H. (2013) Simultaneous estimation of occupancy and detection probabilities: an illustration using Cincinnatian brachiopods. *Paleobiology* 39, 193–213
- Lawing, A. Michelle, et al. (2021). Occupancy models reveal regional differences in detectability and improve relative abundance estimations in fossil pollen assemblages. *Quaternary Science Reviews* 253: 106747.  
<https://doi.org/10.1016/j.quascirev.2020.106747>
- Reitan, T., Ergon, T., & Liow, L. H. (2022). Relative species abundance and population densities of the past: Developing multispecies occupancy models for fossil data. *Paleobiology*, 1-16.  
doi:10.1017/pab.2022.17
- Reitan, T., E. D. Martino, and L. H. Liow. (2024). Estimating relative species abundance using fossil data identified to different taxonomic levels. *Ecography* 2024:e06866.



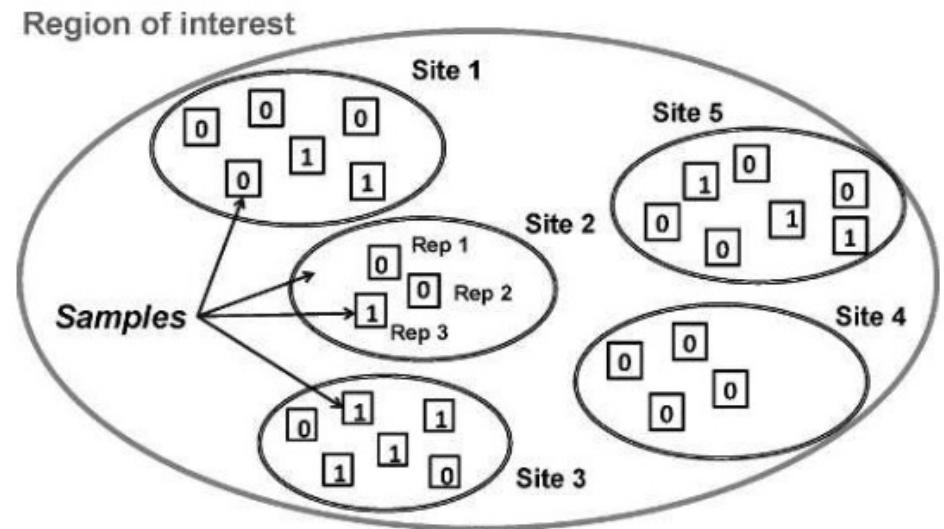


# Occupancy modeling in paleo



$$\Pr(h = 1010) = \varphi(1 - p)^2 p^2$$

$$\Pr(h = 0000) = \text{?????}$$



$$\Pr(h = 1010) = \varphi(1 - p)^2 p^2$$

$$\Pr(h = 0000) = (1 - \varphi) + \varphi(1 - p)^4$$

$$L(\varphi, p | h_1, h_2, \dots, h_s) = \prod_{i=1}^s \Pr(h_i)$$

$$\Pr(h = 1010) = \varphi(1 - p)^2 p^2$$

$$\Pr(h = 0000) = (1 - \varphi) + \varphi(1 - p)^4$$

$$L(\varphi, p | h_1, h_2, \dots, h_s) = \prod_{i=1}^s \Pr(h_i)$$

# R package occupancy modeling

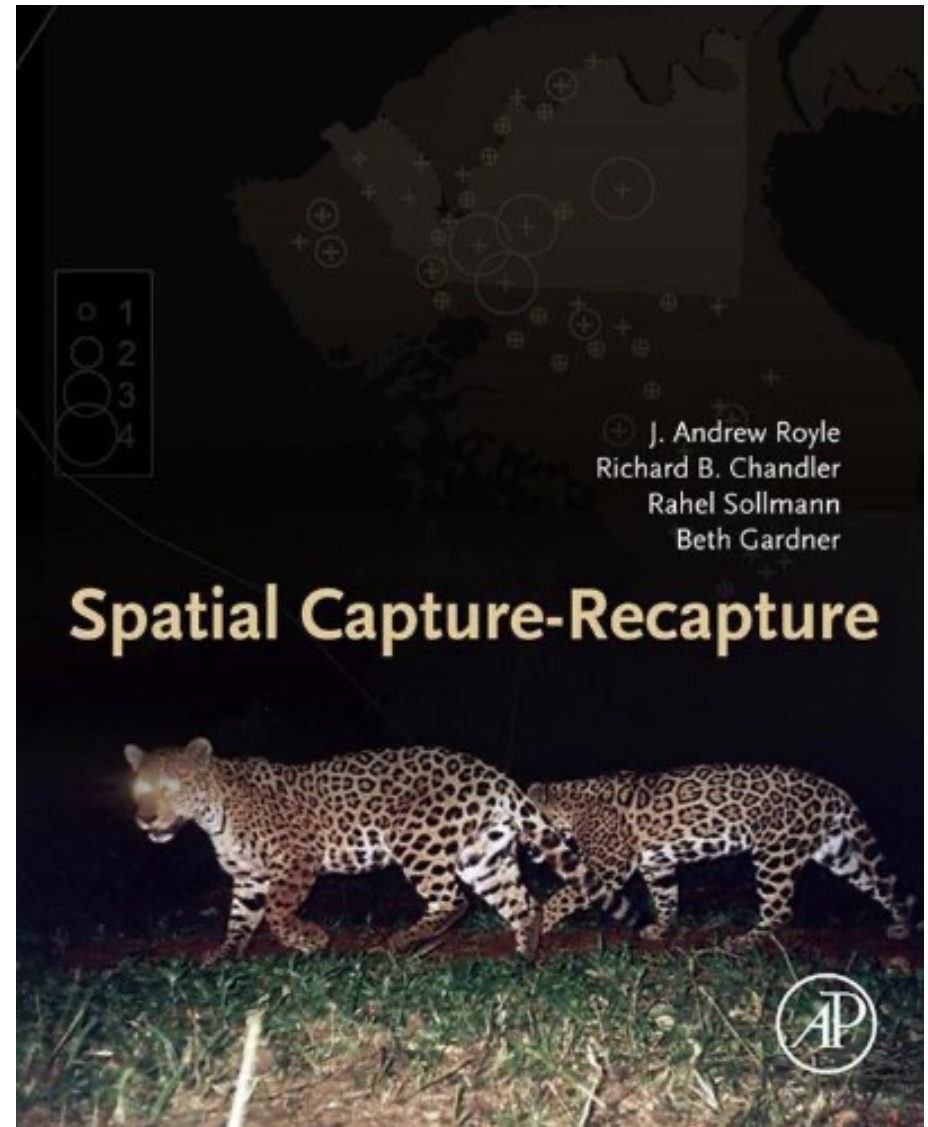
- openCR, serc
- unmarked

# Some resources for occupancy modeling

- <https://kevintshoemaker.github.io/NRES-746/Occupancy.html>
- <https://science.uct.ac.za/seec/stats-toolbox-seminars-spatial-and-species-distribution-toolboxes/single-season-occupancy-models-using-bayesian-approach>

# Spatial capture recapture

- Combining the best of capture recapture and occupancy
- Also implemented in serc, openCR, unmarked



Brief comparisons of CMR with other approaches (paleo context)

# HISTORY OF ESTIMATION TAXONOMIC RICHNESS AND DIVERSIFICATION RATES

Nichols & Pollock 1983 Estimating taxonomic diversity, extinction rates, and speciation rates from fossil data using capture-recapture models. *Paleobiology* 9, 150–163

Foote & Raup 1996 Fossil preservation and the stratigraphic ranges of taxa. *Paleobiology*

Foote 1999/2001 (Boundary crossers method)

Alroy et al. 2001 (sampling standardization)

Connolly and Miller papers 2001-2 using CMR (Connolly is an ecologist)

Foote 2003 (few people use this) – CMR-like, but accounts for origination and extinction within time interval (but see robust design)

(2009) My own first capture recapture paleo-paper – I met Nichols in 2006; short course paper with Nichols

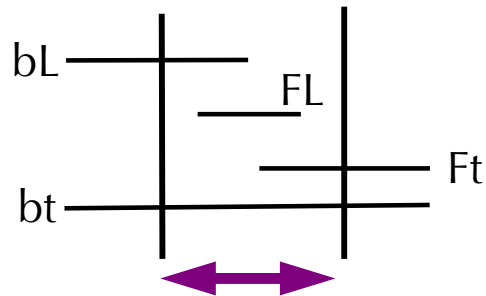
Silvestro, Schinitzler & Liow Syst bio 2014 Pyrate model paper (not the software)

Warnock et al. 2020 RevBayes (starting from birth death models but dropping the “relationships”)



# Comparisons with other approaches

Foote's per capita origination and extinction rates (boundary crossers)



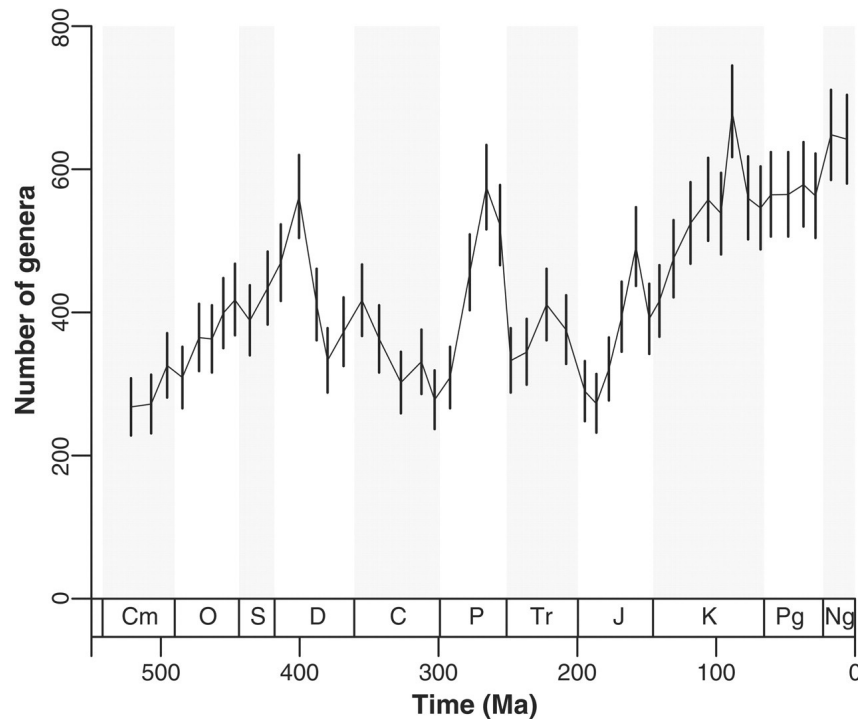
- Easy to use
- Lose information of “singletons” (FL class information not used)
- Hard to compare different models

$$\hat{p} = \ln \left( \frac{N_t}{N_{bt}} \right) / \Delta t$$

$$\hat{q} = \ln \left( \frac{N_b}{N_{bt}} \right) / \Delta t$$

# Comparisons with other approaches

Sampling standardization approaches (including SQS)



- Easy to use
- Assume that even sample or quorums will allow unbiased relative change to be estimated
- Ad hoc rather than modelling approach (preservation is not modelled although it is part of the process)
- Hard to compare different models

# Comparisons with other approaches

CMR-like approaches in paleo independently developed

Foote 2003 Journal of Geology

Alroy “three-timer” and related methods

- Some hard, some easy to use, not easy to understand
- Hard to compare different models
- (very) Special cases of CMR

# Comparisons with other approaches

PyRate (Silvestro et al. Sys Bio 2014)

Key preservation assumption (different) is the shape of species observations (beta distribution based on “hat”)

Smooths out temporal information

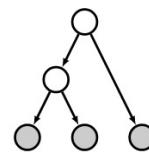
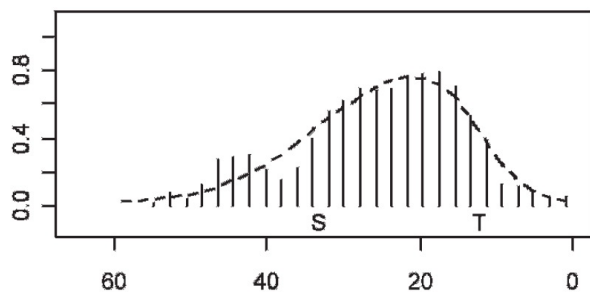
Conditioned on at least one observation per taxon (like the CMR models conditioned on first observation)

RevBayes

[https://revbayes.github.io/tutorials/fbd\\_range/](https://revbayes.github.io/tutorials/fbd_range/)

Rachel’s lecture!

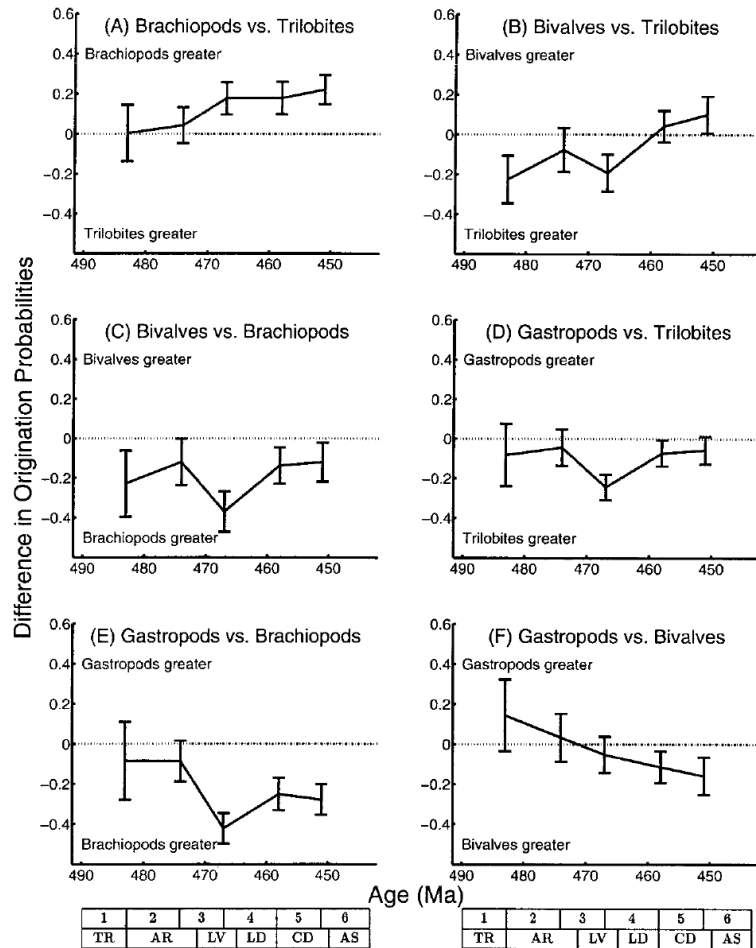
**Discoaster deflandrei (1249)**



**RevBayes**

Bayesian phylogenetic inference using probabilistic graphical models and an interpreted language

# Some paleo papers using CMR



Nichols, J.D. and Pollock, K.H. (1983) Estimating taxonomic diversity, extinction rates, and speciation rates from fossil data using capture-recapture models. *Paleobiology* 9, 150–163

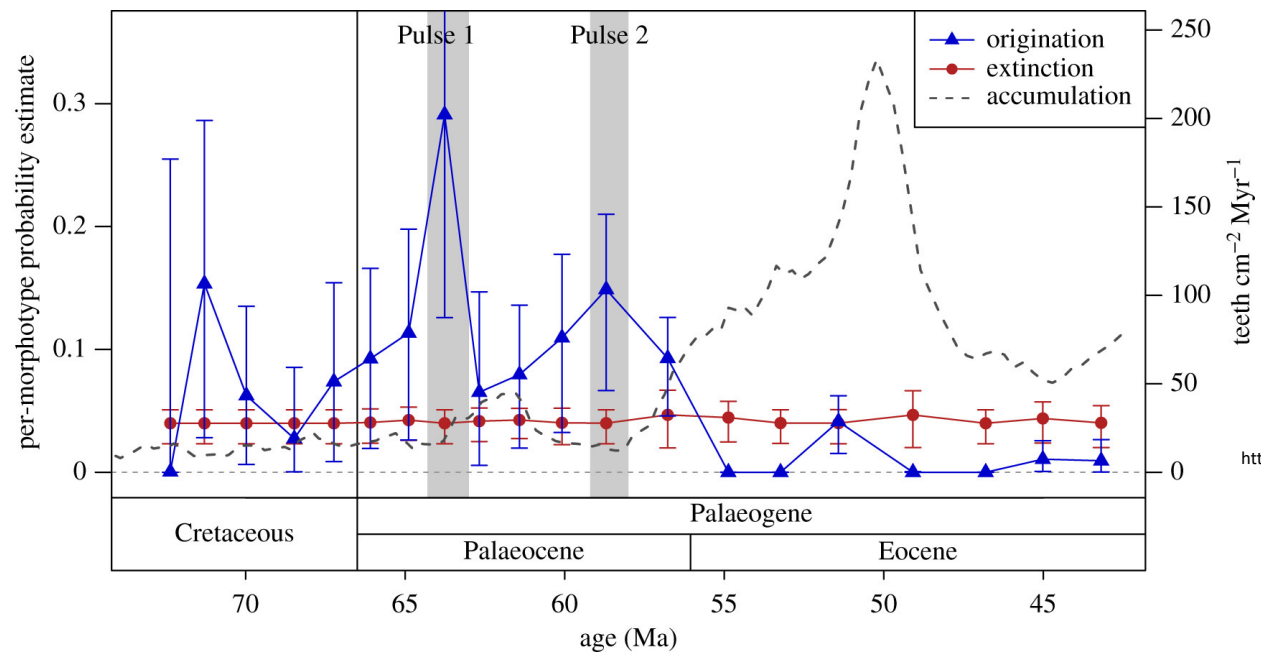
Connolly, S.R. and Miller, A.I. (2001) Joint estimation of sampling and turnover rates from fossil databases: Capture-Mark-Recapture methods revisited. *Paleobiology* 27, 751–767

Connolly, S.R. and Miller, A.I. (2001) Global Ordovician faunal transitions in the marine benthos: proximate causes. *Paleobiology* 27, 779–795

Connolly, S.R. and Miller, A.I. (2002) Global Ordovician faunal transitions in the marine benthos: ultimate causes. *Paleobiology* 28, 26–40

Fig. 4. Model-averaged estimates of between-class differences in genus origination probabilities over time ob-

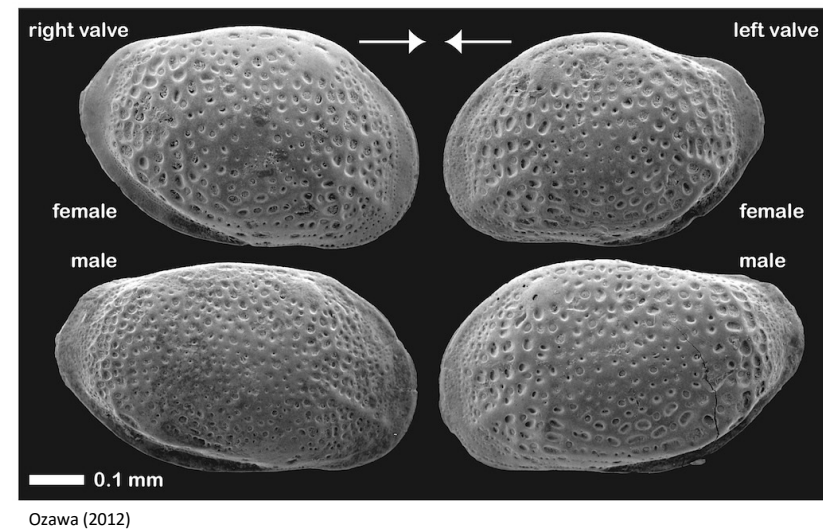
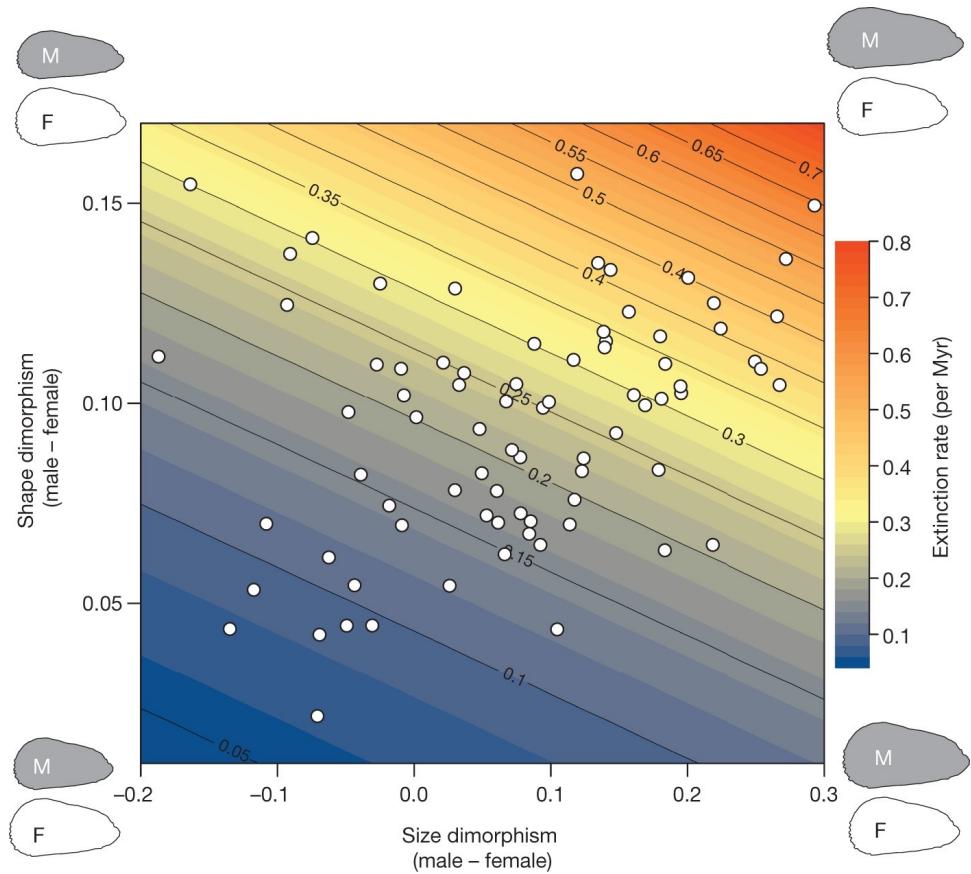
# Some paleo papers using CMR



<https://news.harvard.edu/gazette/story/2018/11/tiny-teeth-tell-the-story-of-two-fish-species-rapid-evolution/>

Sibert, E. *et al.* (2018) Two pulses of morphological diversification in Pacific pelagic fishes following the Cretaceous - Palaeogene mass extinction. *Proc. R. Soc. B-BIOLOGICAL Sci.* 285,

# Some paleo papers using CMR



Martins, M.J.F. *et al.* (2018) High male sexual investment as a driver of extinction in fossil ostracods. *Nature*

# Some paleo papers using CMR

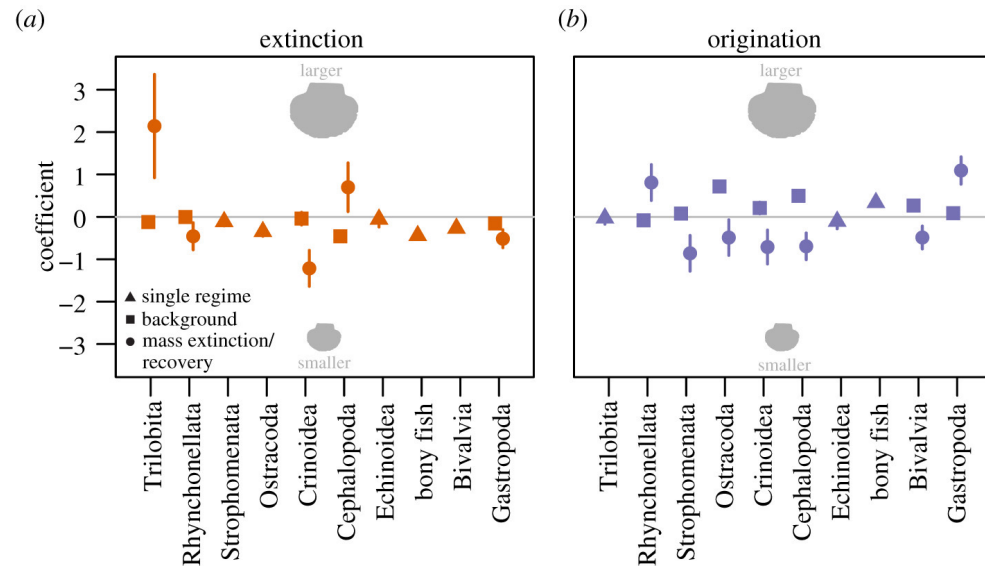
Payne, J., & Heim, N. (2020). Body size, sampling completeness, and extinction risk in the marine fossil record. *Paleobiology*, 46(1), 23-40. doi:10.1017/pab.2019.43

Pedro M. Monarrez, Noel A. Heim and Jonathan L. Payne 2021

[Mass extinctions alter extinction and origination dynamics with respect to body size](#)

Proc B

“CMR analysis of the fossil record reveals a bias against the sampling of smaller-bodied genera within classes “





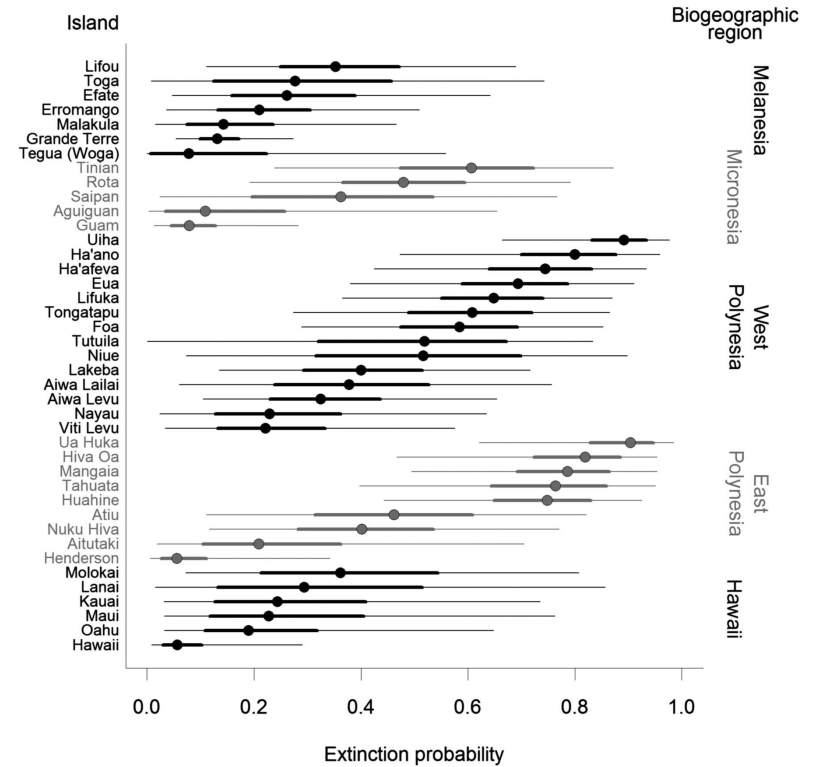
# Some paleo papers using CMR

## Magnitude and variation of prehistoric bird extinctions in the Pacific 2013

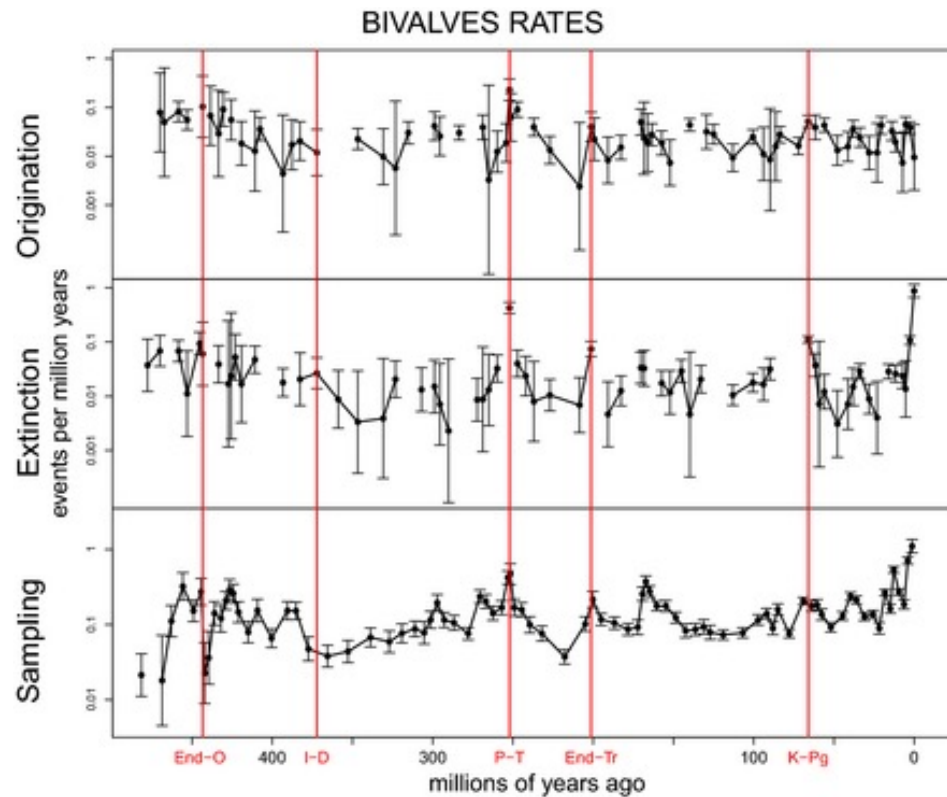
[Richard P. Duncan](mailto:richard.duncan@canberra.edu.au), [Alison G. Boyer](mailto:alison.g.boyer@canberra.edu.au), and [Tim M. Blackburn](mailto:tim.m.blackburn@canberra.edu.au)

PNAS

“We use a Bayesian mark-recapture approach to model gaps in the fossil record and to quantify losses of nonpasserine landbirds on 41 Pacific islands.”



# Some paleo papers using CMR



Liow, L.H. *et al.* (2015) Ecological interactions on macroevolutionary time scales: clams and brachiopods are more than ships that pass in the night. *Ecol. Lett.* 18, 1030–1039